Math 115

Summer 2017

Lecture 8

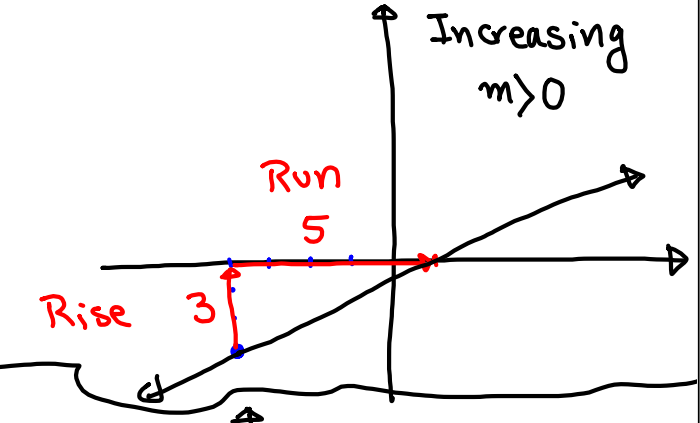
Graph using point & slope:

$y+3=\frac{3}{5}(x+4)$

$y-y_1=m(x-x_1)$

Point $(-4,-3)$

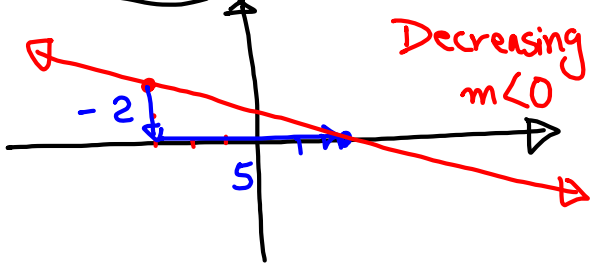
Slope $m=\frac{3}{5}$



$y-2=\frac{-2}{5}(x+3)$

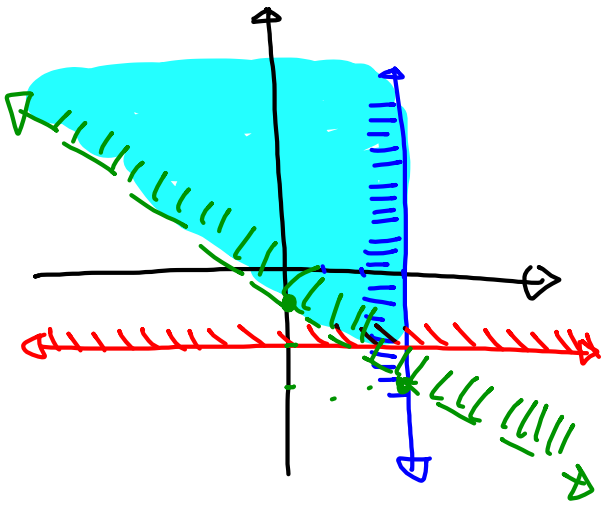
Point $(-3,2)$

$m=\frac{-2}{5}$



Graph & Shade:

$$\begin{cases} x \le 3 \\ y \ge -2 \\ y > \frac{-2}{3}x - 1 \end{cases}$$



How to find eqn of a line with one point & slope:

(x_1, y_1)

① when $m = 0 \Rightarrow y = y_1$

② when m is undefined $\Rightarrow x = x_1$
"No slope"

③ otherwise $\Rightarrow$ use $y - y_1 = m(x - x_1)$
Simplify,
final ans in $y = mx + b$

Find eqn of a line that contains $(-2,4)$ with

1) $m=0$

H.L.

$\boxed{y=4}$

2) undefined slope

V.L.

$\boxed{x=-2}$

3) $m=3$

$y-y_1=m(x-x_1)$

$y-4=3(x--2)$

$y-4=3(x+2)$

$\boxed{y=3x+10}$

4) $m=\frac{-1}{2}$

$y-y_1=m(x-x_1)$

$y-4=\frac{-1}{2}(x--2)$

$y-4=\frac{-1}{2}x-1$

$\boxed{y=\frac{-1}{2}x+3}$

Find eqn of a line that contains $(0,-5)$ with

1) Zero Slope

$m=0$

H.L.

$\boxed{y=-5}$

2) No slope

undefined slope

V.L. $\boxed{x=0}$

3) $m=4$

$y-y_1=m(x-x_1)$

$y--5=4(x-0)$

$y+5=4x$

$\boxed{y=4x-5}$

4) $m=\frac{-2}{3}$

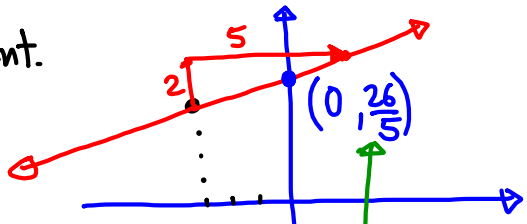
$y-y_1=m(x-x_1)$

$y--5=\frac{-2}{3}(x-0)$

$\boxed{y=\frac{-2}{3}x-5}$

Graph a line that contains $(-3,4)$ with slope $\frac{2}{5}$.

Find its eqn in slope-Int. form.



$y - y_1 = m(x - x_1)$

$y - 4 = \frac{2}{5}(x - -3)$

$y - 4 = \frac{2}{5}(x+3)$

$5y - 20 = 2(x+3)$

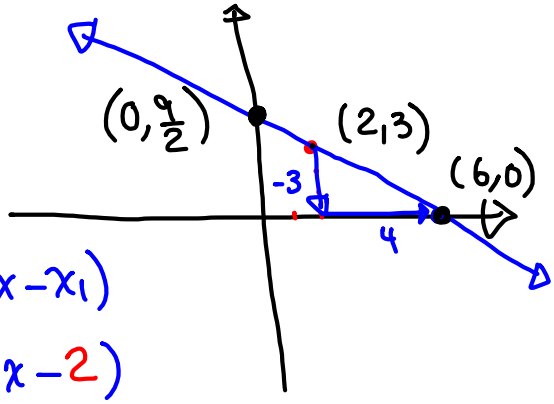
$5y - 20 = 2x + 6$

$5y = 2x + 26$

$\boxed{y = \frac{2}{5}x + \frac{26}{5}}$ slope-Int. form

Graph a line that contains $(2, 3)$ with slope $\frac{-3}{4}$.

Find its eqn in slope-Int. form.



$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-3}{4}(x - 2)$

$4y - 12 = -3(x-2)$

$4y - 12 = -3x + 6$

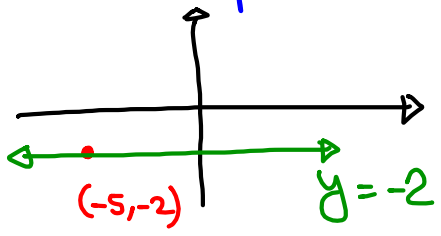
$4y = -3x + 18$

$y = \frac{-3}{4}x + \frac{18}{4}$

$\boxed{y = \frac{-3}{4}x + \frac{9}{2}}$

Graph a line that contains $(-5, -2)$ with zero slope. Find its equation.

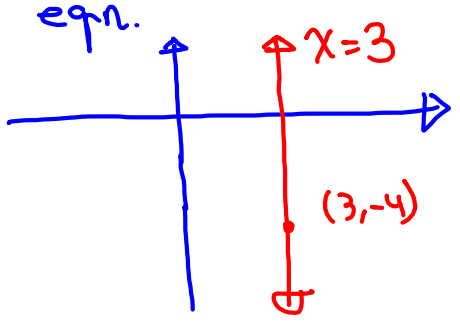
H.L.



Graph a line that contains $(3, -4)$ with undefined slope. Find its eqn.

NO slope

V.L.



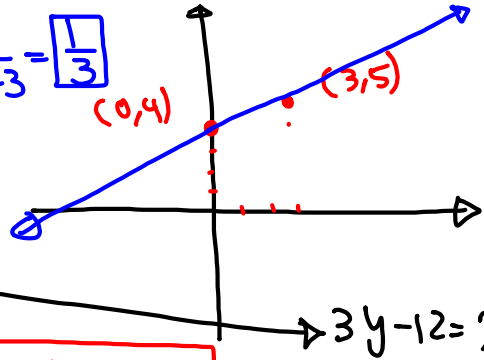
Find eqn of a line that contains $(0, 4)$ and $(3, 5)$. Graph it as well.

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4-5}{0-3} = \frac{-1}{-3} = \boxed{\frac{1}{3}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{3}(x - 0)$$

$$y - 4 = \frac{1}{3}x \Rightarrow \boxed{y = \frac{1}{3}x + 4}$$



$3y - 12 = x$

$3y = x + 12$

$y = \frac{1}{3}x + \frac{12}{3}$

$\boxed{y = \frac{1}{3}x + 4}$

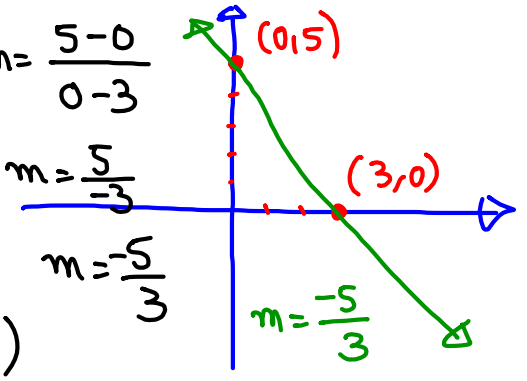
Find the eqn of a line that contains $(0,5)$ & $(3,0)$. write the ans in Slope-Int form.

Graph the line & Show Rise & Run of its slope.

$m = \frac{5-0}{0-3}$

$m = \frac{5}{-3}$

$m = \frac{-5}{3}$



$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-5}{3}(x - 0)$

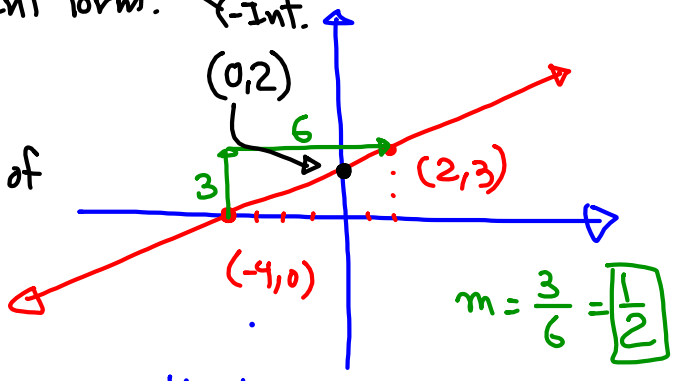
$y - 5 = \frac{-5}{3}x \Rightarrow \boxed{y = \frac{-5}{3}x + 5}$

Find eqn of a line that contains $(-4,0)$ and $(2,3)$ in Slope-Int form.

Graph the line, Show Rise & Run of its slope.

$(-4,0)$ & $(2,3)$

$m = \frac{0-3}{-4-2} = \frac{-3}{-6} = \frac{1}{2}$



$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}(x - 2)$

$y - 3 = \frac{1}{2}x - 1$

$\boxed{y = \frac{1}{2}x + 2}$ Y-Int $(0,2)$

Parallel & Perpendicular Slant lines:

Two Parallel lines $\Leftrightarrow m_1 = m_2$

Two Perpendicular lines $\Leftrightarrow m_1 \cdot m_2 = -1$

$m_1 = 5$, $m_2 = 5$

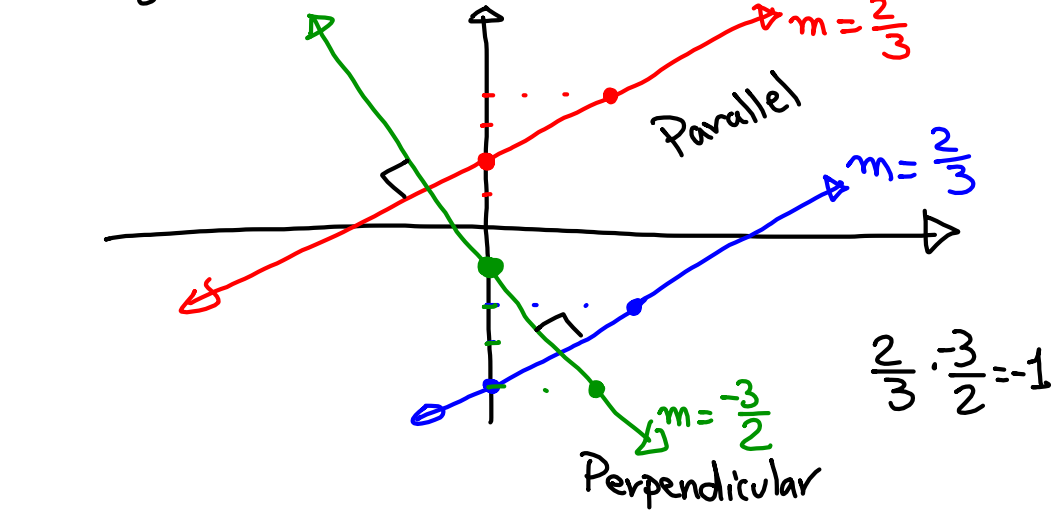
$m_1 = m_2$

Parallel

$m_3 = -\frac{1}{5}$

Perpendicular to both of them

$m_1 \cdot m_2 = 5 \cdot -\frac{1}{5} = -1$



Find eqn of a line that contains (0,4) and is **parallel** to $y = \frac{2}{3}x - 2$.

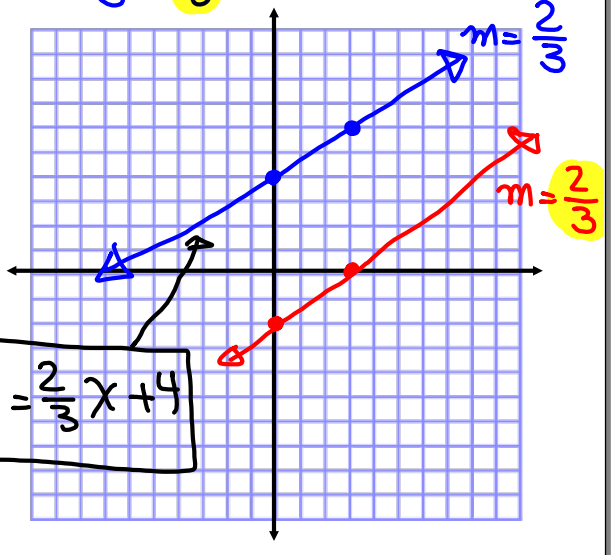
Graph both lines.

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{2}{3}(x - 0)$

$y - 4 = \frac{2}{3}x$

$y = \frac{2}{3}x + 4$



Find eqn of a line that contains (-4, 5) and is **perpendicular** to $y = \frac{1}{2}x + 1$.

Graph both lines.

Solve for m:

$\frac{1}{2} \cdot m = -1$

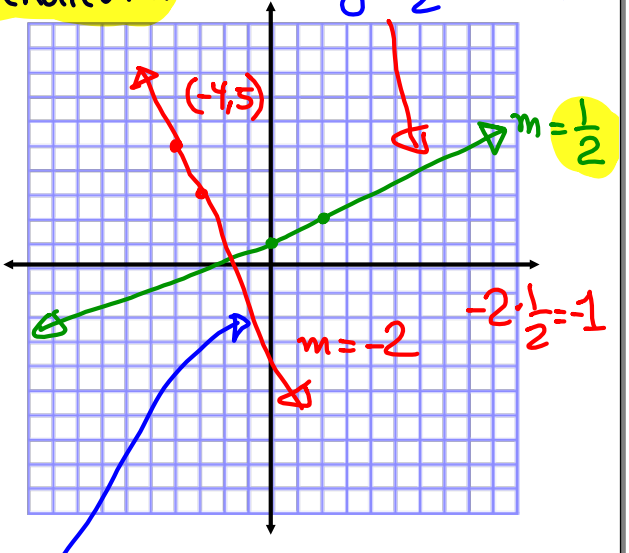
$m = -2 = \frac{-2}{1}$

$y - y_1 = m(x - x_1)$

$y - 5 = -2(x - -4)$

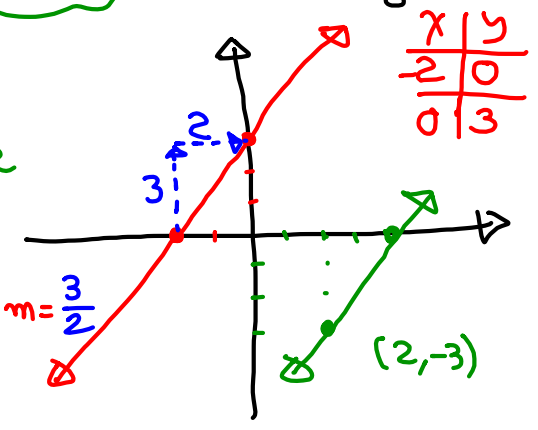
$y - 5 = -2x - 8$

$y = -2x - 3$



Find eqn of a line that contains $(2,-3)$ and is parallel to $3x-2y=-6$.

Graph both lines.



Parallel lines → Same slope

$y-y_1=m(x-x_1)$

$y-(-3)=\frac{3}{2}(x-2)$

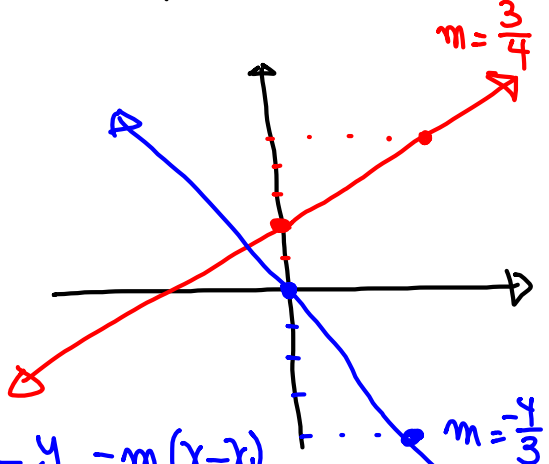
$y+3=\frac{3}{2}(x-2)$

$y+3=\frac{3}{2}x-\frac{3}{2}\cdot 2$

$y=\frac{3}{2}x-6$

Find eqn of a line that contains the Origin and is perpendicular to the line $3x-4y=-8$.

Graph both lines.



$3x-4y=-8$

$-4y=-3x-8$

$y=\frac{3}{4}x+2$

$\frac{3}{4}\cdot m=-1$

$m=\frac{-4}{3}$

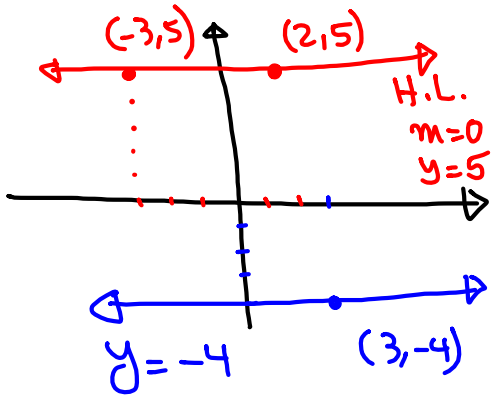
$y-y_1=m(x-x_1)$

$y-0=\frac{-4}{3}(x-0)$

$y=\frac{-4}{3}x$

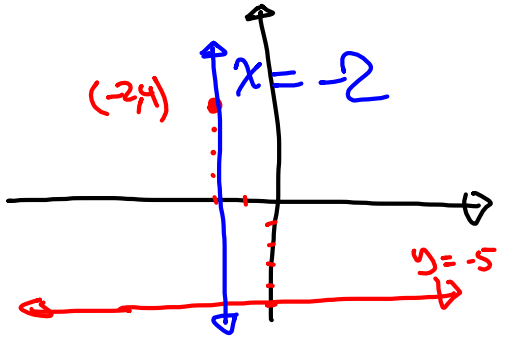
Find eqn of a line that contains (3,-4) and is parallel to a line that contains (-3,5) and (2,5).

Graph both lines.



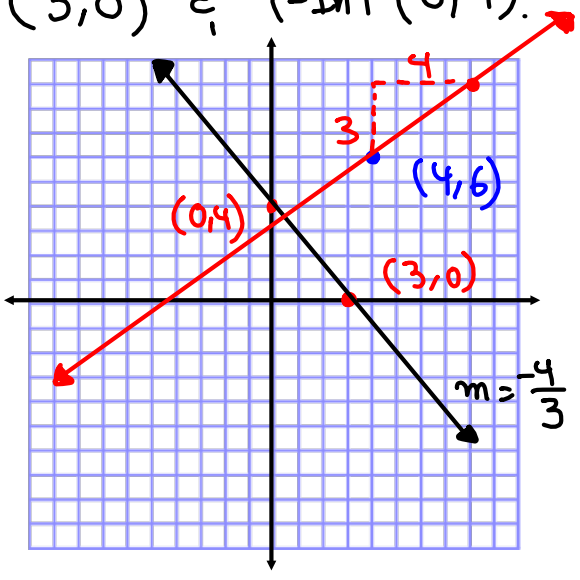
Find eqn of a line that perpendicular to the line $y=-5$ and it contains the point (-2,4).

Graph both lines.



Find the eqn of a line that contains (4,6) and is perpendicular to the line with x-Int (3,0) & y-Int (0,4).

Graph both lines.



$-\frac{4}{3} \cdot m = -1$

$m = \frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{3}{4}(x - 4)$

$y = \frac{3}{4}x + 3$

Solving System of linear Equations:

Ch. 8

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

A Solution, if exists, is an ordered-Pair.

(x, y)

Is (0,4) a Solution? Solution should satisfy both eqns.

$0 + 4 = 4$ ✓

$0 - 4 = 2$ ✗

(0,4) is not a Solution.

Is (3,1) a Solution?

$3 + 1 = 4$ ✓ $3 - 1 = 2$ ✓

Yes (3,1) is a Solution.

Is $(3,-2)$ a solution for

$$\begin{cases} 2x-3y=6 \\ y=\frac{2}{3}x+2 \end{cases} ?$$

Checking $2x-3y=6$

$2(3)-3(-2)=6$

$6+6=6$

$12=6$ ✗

$(3,-2)$ is not a solution

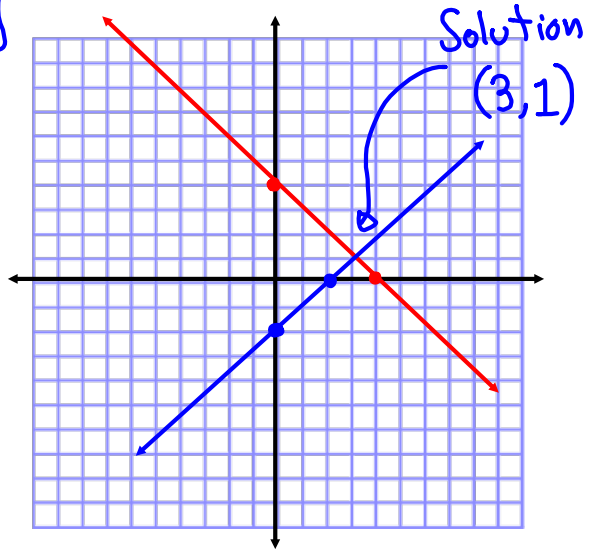
No need to check the second equation since the first eqn failed.

We can solve a system of linear equations in two variables by

- Graphing Method
- Substitution Method
- Elimination Method

Solve by graphing

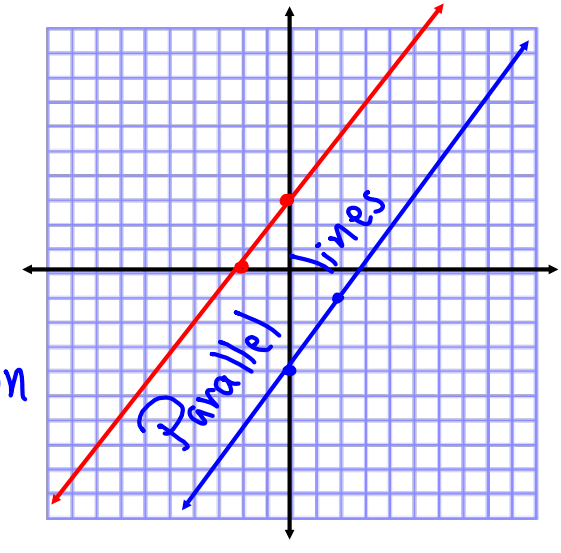
$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$



Solve by graphing

$$\begin{cases} 3x - 2y = -6 \\ y = \frac{3}{2}x - 4 \end{cases}$$

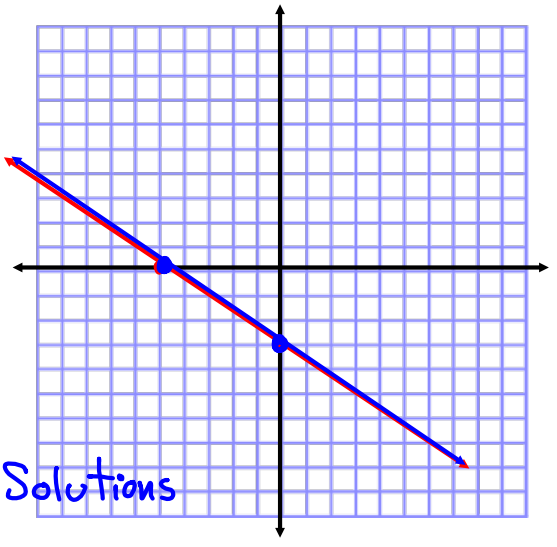
$\emptyset$ No Solution



Solve by Graphing

$$\begin{cases} 3x + 5y = -15 \\ y = \frac{-3}{5}x - 3 \end{cases}$$

$\frac{-3}{5} = \frac{3}{-5}$



Infinitely Many Solutions

Solve by Substitution

$$\begin{cases} 3x - 2y = 5 \\ y = x - 3 \end{cases} \Rightarrow 3x - 2(x - 3) = 5$$

$x = -1$

$y = -1 - 3$

$y = -4$

$(-1, -4)$

$$\begin{cases} 2x + 3y = 5 \\ y = \frac{1}{3}x - 1 \end{cases}$$

$2x + 3\left(\frac{1}{3}x - 1\right) = 5$

Solve by Subs. Method.

$2x + x - 3 = 5$

$3x = 8$

$\boxed{x = \frac{8}{3}}$

$y = \frac{1}{3}\left(\frac{8}{3}\right) - 1$

$y = \frac{8}{9} - 1$ $\quad y = \frac{8}{9} - \frac{9}{9} = \frac{8-9}{9} = \frac{-1}{9}$ $\quad \boxed{y = \frac{-1}{9}}$

$\left(\frac{8}{3}, \frac{-1}{9}\right)$

Solve by Subs. Method

$$\begin{cases} 3x + 4y = 8 \\ y = \frac{-3}{4}x - 2 \end{cases}$$

$3x + 4\left(\frac{-3}{4}x - 2\right) = 8$

$3x - 3x - 8 = 8$

$-8 = 8$

false

$\emptyset$

Solve by elimination (Addition) method

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

$2x = 6$

$\boxed{x = 3}$

$3 + y = 4$

$\boxed{y = 1}$

$(3, 1)$

Solve by elimination:

$$\begin{cases} 3x + 2y = 7 \\ 2\left\{ x - y = 9 \right. \end{cases} \Rightarrow \begin{cases} 3x + 2y = 7 \\ 2x - 2y = 18 \end{cases}$$

$5x = 25$

$\boxed{x = 5}$

$5 - y = 9$

$\boxed{y = -4}$

$(5, -4)$

$$\begin{array}{l} -2\{\ 2x + 3y = 4 \\ 3\{\ 5x + 2y = 10 \end{array} \Rightarrow \begin{array}{l} -4x - 6y = -8 \\ 15x + 6y = 30 \end{array}$$

$11x = 22$

$x = 2$

$2(2) + 3y = 4$

$4 + 3y = 4$

$3y = 4 - 4$

$3y = 0$

$y = \frac{0}{3}$

$y = 0$

$(2, 0)$

Solve by elimination

$$\begin{array}{l} 2\{\ 3x + 5y = -7 \\ 5\{\ 4x - 2y = 8 \end{array} \Rightarrow \begin{array}{l} 6x + 10y = -14 \\ 20x - 10y = 40 \end{array}$$

$26x = 26$

$x = 1$

$3(1) + 5y = -7$

$5y = -10$

$y = -2$

$(1, -2)$

Class Quiz

1) Graph $5x - 3y = -15$

2) Graph $y = \frac{-3}{4}x + 3$

Study Guides 9 & 10 are due Monday.

Graphing Project Points & Line due Tuesday.